VIBRATIONS AND BIFURCATIONS OF RELAXING HEAT FLUXES IN NONLINEAR MEDIA

O. N. Shablovskii

UDC 536.2.01

The objects of investigation are nonlinear media possessing a thermal memory. Vibrational-relaxational thermal regimes for one-dimensional fields with three types of spatial symmetry are studied. Examples of bifurcation processes for one-dimensional and self-similar two-dimensional variants are given.

Analysis of the formation of relaxation structures of thermal fields occupies an important place in the problem of investigation of transfer processes in locally nonequilibrium systems. The most significant results of the studies in this area are presented in reviews [1-4].

As is known (see detailed bibliography in [1-4], the Maxwell relaxational model of heat transfer in a stationary medium is composed of a heat-flux equation and an energy equation:

$$\mathbf{q} + \gamma \frac{\partial \mathbf{q}}{\partial t} = - \lambda \operatorname{grad} T , \quad c \frac{\partial T}{\partial t} + \operatorname{div} \mathbf{q} = q_v . \tag{1}$$

In a one-dimensional case with a plane, cylindrical, and spherical symmetries this system is written in the form

$$q + \gamma \frac{\partial q}{\partial t} = - \lambda \frac{\partial T}{\partial x} , \quad c \frac{\partial T}{\partial t} + \frac{\partial q}{\partial x} + \frac{\nu q}{x} = q_v , \quad \nu = 0, 1, 2 . \tag{2}$$

On the basis of model (2) studies have been made of boundary-layer transitions [5], thermal shock waves [2, 6-8], phase interfaces [9], and dynamic thermal hysteresis [10].

The objectives of the present work are 1) determination of the conditions for the onset of oscillations of temperature and of heat flux in nonlinear media in the presence of volume sources (sinks) of energy; and 2) study of bifurcation phenomena in one-dimensional and self-similar two-dimensional thermal fields.

1. **Vibrational-Relaxational Processes.** We will take the following relations:

$$\begin{gathered} \lambda = \lambda_0 T^{n_1} , \quad c = c_0 T^{n_2} , \quad \gamma \equiv \text{const} , \quad \beta = (n_1 - n_2)/(1 + n_2) , \\ q_v = Q_0 T^{1+n_2} , \quad \tau = \exp(-kt) , \quad \gamma k n = 1 , \quad m_2 = 1 + n_2 > 0 , \\ H m_2 + 1 = n , \quad Q_0 = - H k c_0 , \quad n_0 = H (n_2 - n_1) + 2 , \\ T = \tau^H \Theta , \quad q = k \tau^n v , \quad m_2 U = c_0 \Theta^{m_2} , \quad a_0 c_0 = \lambda_0 (m_2/c_0)^{\beta} . \end{gathered} \tag{3}$$

Then, heat-transfer equations (2) take the form

$$U_\tau = v_x + \frac{\nu v}{x} , \quad k^2 \gamma v_\tau = \left[\tau^{-n_0} \int W dU \right]_x , \quad W = a_0 U^{\beta} , \tag{4}$$

which is similar to the gas dynamic equations for nonadiabatic flow in Lagrange coordinates [11].

To make the equations dimensionless, we use scales that admit invariance of the dimensional and dimensionless forms of representation. For example, $g_b = \lambda_b T_b / x_b$, $\lambda_b = c_b x_b^2 / t_b$, etc.

P. O. Sukhoi Gomel Polytechnical Institute, Belarus. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 70, No. 1, pp. 123-129, January-February, 1997. Original article submitted January 21, 1994.

1062-0125/97/7001-0125$18.00 ©1997 Plenum Publishing Corporation

Now, we construct a solution having the same structure as for a gas flow with a linear velocity profile [12]. According to the first equation in system (4), we have: $x^\nu U = \psi_x$, $x^\nu v = \psi_\tau$. We take $x^\nu v = \psi L(\tau)$, then $v_\tau = v(L + \dot{L}L^{-1})$. From this we obtain

$$v = V(x)\, B(\tau)\,, \quad B(\tau) = \exp\left[\int (L + \dot{L}L^{-1})\, d\tau\right], \quad B = \dot{C}(\tau)\,,$$

$$U = \left[\dot{V}(x) + \nu V x^{-1}\right] C(\tau)\,, \quad B = CL\,, \quad \psi = x^\nu v/L\,.$$

Substituting these relations into the second equation of system (4) and separating the variables, we obtain two ordinary differential equations for the functions $C(\tau)$ and $V(x)$:

$$\tau^{n_0} \frac{d^2 C}{d\tau^2} = \mu C^{1+\beta}\,, \quad \mu \equiv \text{const} \neq 0\,, \tag{5}$$

$$\mu V = \frac{a_0}{k^2 \gamma} \left(\frac{dV}{dx} + \frac{\nu V}{x}\right)^\beta \frac{d}{dx}\left(\frac{dV}{dx} + \frac{\nu V}{x}\right). \tag{6}$$

We consider some of the properties of the solutions of Eqs. (5) and (6). We note that Eq. (5) represents the Émden–Fowler equation [13]; mathematically it has been studied for particular values of β and n_0 that have no interesting applications to heat transfer. For Eq. (5) we take the solution

$$C \equiv C_0 = C_{00} \exp(H_1 t)\,,$$

$$C_{00}^\beta = H_1 (k + H_1)/(k^2 \mu)\,, \quad H_1 = Hkm_2\,, \quad k > 0\,. \tag{7}$$

Formula (7) gives a time-independent temperature distribution over x. Then, we linearize Eq. (5) near (7): $C = C_0 + C_1$, $|C_1| \ll 1$ and return to the argument t:

$$\frac{d^2 C_1}{dt^2} + k \frac{dC_1}{dt} + k_*^2 C_1 = 0\,, \quad k_*^2 = -H_1 (k + H_1)(1 + \beta) > 0\,. \tag{8}$$

Written in the above form, Eq. (8) is an equation of damped oscillations of a point. Analysis shows that there is a periodic solution:

$$C_1 = C_{10} \exp(-kt/2) \sin\left[\frac{kt}{2}\sqrt{\left(\frac{1}{\delta} - 1\right)} + \varphi_0\right],$$

if the parameter H, characterizing the volume source, is equal to

$$H = \left\{-1 + [1 - \delta^{-1}(1 + \beta)^{-1}]^{1/2}\right\}(2m_2)^{-1}$$

and, moreover, if the following conditions are satisfied: 1) the volumetric heat sink, $Q_0 < 0$, $\mu > 0$, $\delta \in (0, 1)$, $n_1 < -1$, $n_2 > -1$, i.e., $\beta + 1 < 0$; $\nu = 1, 2$; 2) the volumetric liberation of heat, $Q_0 > 0$, $\mu < 0$, $\delta \in (1/[1 + \beta], 1)$, $-1 < n_2 < n_1$, i.e., $\beta + 1 > 0$; $\nu = 0$. If $\delta \geq 1$, the temperatures variation with time is aperiodic.

The solution of Eq. (6) at $\nu = 0$ is represented in parametric form by using the quadrature

$$x(y) = \int_0^y p^{-1/2} dy\,, \quad p = 2K + m_2\,, \quad m_0 = \mu k \frac{1 + \beta}{a_0 n}\,, \quad b = \frac{\beta + 2}{\beta + 1}\,, \quad \beta = 2n_3\,,$$

$$m_0 (2 + n_3) K = (1 + n_3)\left[(m_0 y + m_1)^b - m_1^b\right], \quad 2K(y_1) + m_2 = 0\,, \quad m_0 y_1 = -m_1\,,$$

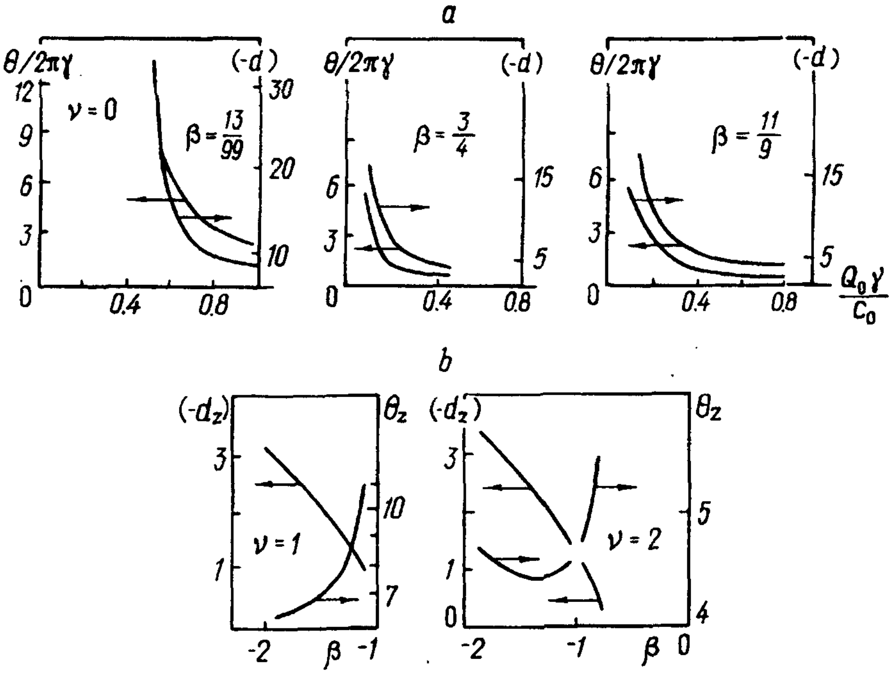

Fig. 1. Period and decrement of oscillations in time (plane case) (a) and over the logarithmic radial coordinate (cylindrical and spherical symmetries) (b).

$$V(x) \equiv p(y), \quad dV/dx = (m_0 y + m_1)^{1/(\beta+1)}, \quad x \in [0, \infty), \quad y \in [0, y_1).$$

Here, the following conditions are satisfied: $x \to \infty$, $u \to 0$, $q \to 0$, when $\beta + 1 > 0$. And this means that in the plane case the oscillations of the temperature and heat flux over time occur with volumetric liberation of heat, whereas in the case of heat absorption at $\nu = 0$, there are no oscillations in t. Figure 1a shows dependences of the source parameter $Q_0\gamma/c_0$ of the dimensionless oscillation period $\theta/2\pi\gamma$ and of the logarithmic decrement d on the parameter for three values of β. We also indicate that the dependence of the oscillation period on $\beta > 0$ is monotone increasing; for example, when $n_2 = 1$, $\beta \in (0, 2]$, we have $\theta/2\pi\gamma \in (4.8)$, $\ddot{\theta}(\beta) < 0$.

We take $z = \ln x$, $V = x^M D(z)$, and $M = 1 + (2/\beta) < 0$ for cylindrical and spherical symmetries and represent Eq. (6) as:

$$\frac{dD}{dz} = G - (M + \nu)\, D, \tag{9}$$

$$\frac{dG}{dz} = \frac{\mu}{a_1} G D^{-\beta} - (M - 1)\, G, \quad a_1 = \frac{a_0 n}{k}.$$

The particular solution

$$D_0 = G_0/(M + \nu), \quad G_0^{\beta} = \mu/[a_1 (M + \nu)(M - 1)] > 0, \quad G_0 > 0, \quad M < 0 \tag{10}$$

corresponds to an exponential temperature distribution over the radial coordinate: $T \sim x^{2/(n_1 - n_2)}$. We linearize system (9) in the vicinity of (10), $D = D_0 + D_1$, $G = G_0 + G_1$, $|D_1| \ll 1$, $|G_1| \ll 1$ and obtain the second-order differential equation

$$\frac{d^2 D_1}{dz^2} + M_1 \frac{dD_1}{dz} + 2(M + \nu)\, D_1 = 0, \quad M_1 = M + \nu + 2(1 + \beta)\beta^{-1},$$

which has a solution periodic in z if 1) $\nu = 1$, $-1 < M < 0$ and 2) $\nu = 2$, $-3/2 < M < 0$;

$$D_1 = D_{10} \exp(-M_1 z/2) \sin\left[z\left\{2(M+\nu) - (M_1^2/4)\right\}^{1/2} + \varphi_1\right], \quad z \ge 0.$$

From this we conclude that in the cylindrical case with $\beta \in (-2, -1)$ and a volumetric heat sink, the temperature field oscillates both in time and over the coordinate $z = \ln x$. In the spherical case with $q_v < 0$ at $\beta \in (-1, -0.8)$ the solution has a vibrational character over z, and there are no oscillations in t; with $\beta \in (-2, -1)$ and $q_v < 0$ the situation is similar to the cylindrical case: along with oscillations in t there are oscillations over z. The dependences of the period $\theta_{(z)}$ and of the logarithmic decrement $d_{(z)}$ on the medium nonlinearity parameter β are shown in Fig. 1b; here we must note that in the spherical case the function $\theta_{(z)}(\beta)$ is nonmonotonic. When $\nu = 1, 2$, the properties of the oscillations in t are as follows: the period $\theta/2\pi\gamma$ and the decrement d are monotone increasing functions of the parameter $Q_0\gamma/c_0 < 0$; at $\beta \in (-2, -1)$, we have $\theta/2\pi\gamma \in (10, 20)$, $\dot{\theta}(\beta) > 0$, and $\ddot{\theta}(\beta) > 0$.

For plane symmetry $\nu = 0$ there are no oscillations over the coordinate. For cylindrical and spherical symmetries, oscillations of the thermal field are realized with volumetric absorption of heat, but they are absent, when $q_v > 0$. The period of oscillations over z is independent of the source Q_0; it is determined only by the value of β. For all of the types of symmetry, the period θ decreases rapidly with an increase in the source $|Q_0|$, i.e., an increase in the intensity of the energy source increases the frequency of oscillations.

In an experimental investigation of the thermophysical properties of high-temperature superconductors the effect of thermal relaxation on the result of measurements was noted [14]. The vibrational-relaxational properties of the thermal field obtained in the present work are directly related to this problem. In [14], a plane specimen of high-temperature superconducting yttrium-based ceramic was investigated; continuous heating was provided by a constant-power internal heat source. This fact is in agreement with the result obtained in the present work that in the plane case oscillations in time occur precisely in the case of a volumetric heat supply. Thus, we have conditions under which it is possible to estimate the period of originating oscillations. The necessary condition $-1 < n_2 < n_1$ is satisfied, in particular, for T, K $\in [5, 10]$. For example, in the vicinity of 8 K we have $n_2 \cong 0$ and $n_1 \cong 1$; when $\delta = 0.75$, we obtain $\theta/\gamma \cong 17.2$, and when $\delta = 0.9$, we obtain $\theta/\gamma \cong 31.4$, where $\gamma \sim 500-700$ sec.

2. Inhomogeneous Energy Source and Bifurcation. In the above-mentioned experiments [14], an unambiguous interpretation of the results obtained is hampered by the macrostructural (technological) inhomogeneity of the material. We analyze the case of an inhomogeneous energy source of the form $q_v = k_v x^{2/\beta} B(t)$. For the heat transfer equations we will construct a solution similar in structure to the previous one. We use the notation of Item 1 for those cases in which it will not cause confusion. We represent the solution in the form

$$q = V(x)\, B(t), \quad u = \left(x^{2/\beta} k_v - \frac{dV}{dx} - \frac{\nu V}{x}\right) C(t), \quad B(t) = \dot{C}(t)$$

and after separation of variables we have

$$\gamma \frac{d^2 C}{dt^2} + \frac{dC}{dt} + \mu C^{1+\beta} = 0, \quad \mu \ne 0, \tag{11}$$

$$\mu V = a_0 \left[x^{2/\beta} k_v - \frac{dV}{dx} - \frac{\nu V}{x}\right]^{\beta} \frac{d}{dx}\left[x^{2/\beta} k_v - \frac{dV}{dx} - \frac{\nu V}{x}\right]. \tag{12}$$

Equation (11) is similar to Eq. (5). We consider Eq. (12). Taking $V = x^M D(z)$ and $z = \ln x$, we obtain:

$$\frac{dD}{dz} = k_v - D(M+\nu) - G, \quad \frac{dG}{dz} = \frac{\mu}{a_0} D G^{-\beta} - (M-1) G, \quad M = 1 + 2\beta^{-1}.$$

The equilibrium state of this dynamic system is as follows:

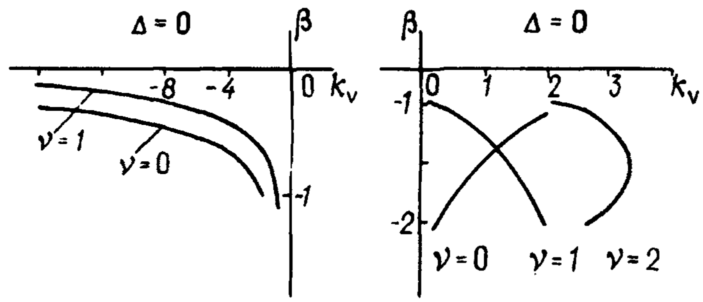

Fig. 2. "Medium–source" system: the boundary of saddles.

$$D_0 = (k_v - G_0)(M + \nu)^{-1}, \quad k_v^0 = G_0 + \frac{a_0}{\mu}(M + \nu)(M - 1) G_0^{1+\beta}. \tag{13}$$

Moreover, we have

$$\sigma = (4/\beta) + \nu + 3,$$

$$\Delta = 2(M + \nu)\beta^{-1}[1 + \beta + G_0 (k_v^0 - G_0)^{-1}].$$

Relation (13) allows us to construct a bifurcation diagram [15], i.e., a curve relating the source parameter k_v to the nonlinearity parameter β. In the plane k_v, G_0 these curves are convex downwards, and at $\beta \in [-2, -1]$ and $\beta \in (-1, 0)$ they lie in the first and fourth quadrants; in each of these variants the smaller $|\beta|$, the higher the diagram is located along the k_v-axis. The bifurcation relationship between k_v^0 and G_0 can be found from the condition of tangency of $k_v = \text{const}$ to the line (13) at the point of the minimum:

$$G_0^\beta = -\mu\,[a_0 (M + \nu)(M - 1)(1 + \beta)]^{-1} > 0, \quad k_v^0 = \beta G_0 (1 + \beta)^{-1}. \tag{14}$$

This value of k_v^0 determines the boundary of the saddles $\Delta = 0$. Figure 2 illustrates the form of the curves $\Delta = 0$ for different types of spatial symmetry; here it should be taken into account that

$$1)\ \nu = 0,\ \mu < 0,\ \beta \in (-1, 0);\ \mu > 0,\ \beta \in [-2, -1);$$

$$2)\ \nu = 1,\ \mu < 0,\ \beta \in (-1, 0);\ \beta \in [-2, -1);$$

$$3)\ \nu = 2,\ \mu < 0,\ \beta \in [-2, -1).$$

This means that there are singular saddle-type points, a node or an unstable node near the line $\Delta = 0$. In the vicinity of two singular points (node O_1 and saddle O_2) there are successive phases: on attainment of the bifurcation value (14), points O_1 and O_2 merge, and a complex singular saddle-node (a stable or unstable node) point is obtained; then the singular points disappear.

Consideration of the stability limit of nodes and focuses $\sigma = 0$ gives $\beta_0 = -4/(\nu + 3)$. In the cylindrical case $\beta_0 = -1$, and there are no nonrough equilibrium states. In the plane case, the sign of Δ is determined by the sign of the expression $4\mu G_0^{4/3} - a_0$, when $\Delta > 0$, the singular point is a center, the solution is periodic over z, and the trajectories of the dynamic system are closed curves. Near $\sigma = 0$ with $\Delta > 0$ either a stable or unstable focus may exist, whereas with $\Delta < 0$ there is a saddle. The spherical variant gives $\beta_0 = -4/5$, the sign of Δ is determined by the sign of the expression $4\mu G_0^{4/5} - a_0$; the behavior of the integral curves is similar to the plane case.

3. Plane Circular Region. We will operate with Eqs. (1) in the polar coordinates r and φ. Let us assume that for the quantities λ, c, q_v dependences (3) are satisfied. In this case, the heat-transfer equations have a particular solution:

$$T^{1+n_2} = \left(\frac{1 + n_2}{c_0}\right) r^{2/\beta} f(\varphi), \quad 0 \le \varphi \le 2\pi, \quad 0 < r^0 \le r \le r^1 < \infty, \quad t \ge 0, \tag{15}$$

$$q_1 = kAr^{1+2/\beta} + kBr^N\tau^n, \quad q_2 = k\hat{A}r^{1+2/\beta} - k\hat{B}r^N\tau^n,$$

$$f^{\beta+1} = F, \quad A(k_0 - 1) = \frac{2}{\beta} a_1 F, \quad N = \alpha + 1 + (2\alpha + 2 - \alpha n_0)\beta^{-1},$$

$$\hat{A} = k_0 \int_0^\varphi f d\varphi - \left(2 + \frac{2}{\beta}\right) \int_0^\varphi A d\varphi + C_1, \quad \hat{B} = (1 + N) \int_0^\varphi B d\varphi + C_2,$$

$$k_0 = (n_0 - 2)/\beta, \quad a_1 = a_0/\gamma k^2. \tag{16}$$

Here α, C_1, C_2 are arbitrary constants, $B(\varphi)$ is an arbitrary periodic function with period $2\pi/l$, $l = 1, 2, 3 \dots$.

In the calculations we use formulas that follow from Eqs. (3) and (16):

$$k_0 = Q_1 \left(Q_1 + \frac{1}{\gamma}\right)^{-1}, \quad k = Q_1 + \frac{1}{\gamma}, \quad n = 1 - k_0,$$

$$Q_1 = Q_0 (1 + n_2)/c_0, \quad \kappa^2 = k_0 (k_0 - 1)/a_1 = - Q_1/a_0 > 0.$$

The function $F(\varphi)$ is determined by the differential equation

$$\frac{d^2F}{d\varphi^2} + \frac{4(\beta + 1)^2}{\beta^2} F = (\beta + 1)\kappa^2 F^{1/(\beta+1)}. \tag{17}$$

Solution (15)-(17) has a physical meaning at any nonzero finite r: it describes the stationary temperature and relaxing heat flux in a plane circular region. If $B(\varphi) \equiv 0$, then we have a stationary thermal field.

The exact solution

$$F_0 = f_0^{\beta+1}, \quad f_0^\beta = \frac{\beta^2\kappa^2}{4(\beta + 1)} \tag{18}$$

characterizes a one-dimensional temperature field with cylindrical symmetry. Linearization of Eq. (17) near the value (18) gives:

$$\frac{d^2(\Delta F)}{d\varphi^2} + \frac{4(\beta + 1)}{\beta}(\Delta F) = 0, \tag{19}$$

$$F = F_0 + \Delta F, \quad |\Delta F| << 1,$$

This equation has a solution periodic in φ if the parameter of the medium nonlinearity β satisfies the condition

$$\frac{4(\beta + 1)}{\beta} = m^2, \quad m = 4, 6, 8, \dots \tag{20}$$

The selection of the even values of m is dictated by the structure of the solutions obtained below. It should be borne in mind that the numerical parameter f_0 characterizes the temperature of the material, while the parameter β characterizes its nonlinear properties in the corresponding temperature range. Consequently, the bifurcation value for the source parameter Q_1 is determined by the expression

$$Q_{1*} = - a_0\kappa_*^2, \quad \kappa_*^2 = 4f_0^\beta(\beta + 1)/\beta^2, \tag{21}$$

with $\beta = 4/(m^2 - 4)$ here and below. If we use the Ostrogradskii criterion as a dimensionless characteristic of the energy source

$$\mathrm{Os} = q_V r^2 \Big/ \left(\int_0^T \lambda dT \right),$$

it turns out that in the class of media (3) its bifurcation value is negative and is dependent only on the nonlinear properties of the medium:

$$\mathrm{Os}_* = -4(\beta+1)^2/\beta^2.$$

We note that mathematically the character of the bifurcation presented here is in its basic features similar to that observed in the hydrodynamic problem of the axisymmetric flow of a viscous incompressible fluid caused by a point source [16].

According to Eqs. (19) and (20), the eigenfunction is equal to $\Delta F = h \sin m\varphi$. Following the method of [17], we construct the solution of Eq. (17) in the form of expansions in the small amplitude h:

$$F = F_{0*} + hF_1 + h^2 F_2 + \cdots, \quad (-Q_1/a_0) \equiv \kappa^2 = \kappa_*^2 + h\kappa_1 + h^2\kappa_2 + \cdots$$

For the coefficients of the first and second approximations we obtain:

$$\kappa_1 = 0, \quad \kappa_2 = 5/[6\beta(\beta+1) f_{0*}^{\beta+2}],$$

$$F_1 = \sin m\varphi, \quad F_2 = \left(\frac{2}{3}\sin^2 m\varphi - \frac{1}{2}\right) \Big/ \left(m^2 \beta F_{0*}\right).$$

Let us write Eq. (17) in another form:

$$\frac{d^2 y}{d\psi^2} + y = \frac{1}{2} y^{1/(\beta+1)}, \quad F = yD, \quad \varphi = \frac{\beta\psi}{2(\beta+1)}, \quad 2D^{\beta/(\beta+1)} = \kappa^2\beta^2/(\beta+1).$$

From here we find the first integral

$$\left(\frac{dy}{d\psi}\right)^2 + y^2 = \frac{(\beta+1)}{(\beta+2)} y^{(\beta+2)/(\beta+1)} + C,$$

which illustrates the location of the integral curves on the phase plane y, $\dot{y}$. The closed lines represent solutions periodic in φ. The equation of the separatrix ($C = 0$) is

$$y_s = y_1 \left[\cos\left(\varphi - \frac{\pi}{4}\right)\right]^{2(\beta+1)/\beta}, \quad y_1 = [(\beta+1)/(\beta+2)]^{(\beta+1)/\beta}.$$

The branches of the separatrix converge at the points (0, 0) and (y_1, 0). In the case of small oscillations near the rest point (y_*, 0) we have

$$C = y_*^2 - \frac{(\beta+1)}{(\beta+2)} y_*^{(\beta+2)/(\beta+1)}, \quad y_* = (1/2)^{(\beta+1)/\beta}.$$

With bifurcation from the point $y = y_*$ a limiting cycle is created; its size increases with an increase in the amplitude h; when $h \to \infty$, the cycle merges with the separatrix loop. Applying, just as in [16], the averaged values

$$\langle F\rangle = \langle y\rangle D, \quad h = y_1 D/2, \quad \langle y\rangle = \frac{1}{2\pi}\int_0^{2\pi} y_s d\varphi,$$

we find an expression for $\langle F\rangle$ at large amplitudes:

$$\langle F\rangle = \frac{1\cdot 3\cdot 5\cdot \ldots \cdot (2p-1)}{1\cdot 2\cdot 3\cdot \ldots \cdot p}\,h\,,\quad p = m^2/4\,.$$

We note also that the bifurcation of the solution (15)-(17) is observed for relaxing radial and transversal heat fluxes as well as in the relaxed (stationary) state.

Results. Damping oscillations of T and q may occur not only in time (the case of plane symmetry), but also over a logarithmic radial coordinate (cylindrical and spherical symmetries). In the case of volumetric energy release, oscillations may occur only in the plane case, and with a heat sink, only for fields with radial symmetry. These facts explain the considerable effect of thermal relaxation observed in the experiments with high-temperature superconductors. It is shown that for relaxing thermal fields with an inhomogeneous volumetric energy source there exists a bifurcation point (a complex state of saddle-node equilibrium). The critical values for the source parameter and the medium nonlinearity parameters are calculated in the region of which bifurcation changes in the thermophysical system occur. A class of thermal fields in a plane circular region is indicated for which there exist bifurcation values (21) of the parameter of the nonlinear volumetric energy sink (the generation of a limit cycle from the separatrix loop).

NOTATION

T, temperature; q, vector of specific heat flux; t, time; x, Cartesian (radial) coordinate; λ, coefficient of thermal conductivity; c, specific volumetric heat capacity; q_v, power of internal heat sources; γ, heat flux relaxation time; β, medium nonlinearity parameter; Os, Ostrogradskii criterion. Indices: b, scales of nondimensionalized quantities; the dot over the sign of the function denotes ordinary differentiation; independent variables in the role of subscripts denote partial differentiation; *, denotes the bifurcation value of the parameter.

REFERENCES

1. Yu. A. Buevich and G. P. Yasnikov, Inzh.-Fiz. Zh., **44**, No. 3, 489-504 (1983).
2. E. I. Levanov and E. N. Sotskii, in: Mathematical Simulation. Nonlinear Differential Equations of Mathematical Physics [in Russian], Moscow (1987), pp. 155-190.
3. D. D. Joseph and L. Preziosi, Reviews of Modern Physics, **61**, No. 1, 41-73 (1989).
4. S. L. Sobolev, Uspekhi Fiz. Nauk, **161**, No. 3, 5-29 (1991).
5. O. N. Shablovskii, Inzh.-Fiz. Zh., **40**, No. 3, 510-517 (1981).
6. E. I. Levanov and E. N. Sotskii. Inzh.-Fiz. Zh., **51**, No. 2, 232-240 (1986).
7. O. N. Shablovskii, Inzh.-Fiz. Zh., **46**, No. 5, 832-836 (1984).
8. O. N. Shablovskii, Inzh.-Fiz. Zh., **48**, No. 1, 129-136 (1985).
9. O. N. Shablovskii, Inzh.-Fiz. Zh., **55**, No. 3, 464-471 (1988).
10. O. N. Shablovskii, Inzh.-Fiz. Zh., **59**, No. 1, 149-155 (1990).
11. L. V. Komarovskii and O. N. Shablovskii, Analytical Investigation of Certain Internal Problems of Nonstationary Gas Dynamics and Heat Transfer [in Russian], Tomsk (1981).
12. B. L. Rozhdestvenskii and N. N. Yanenko, Systems of Quasilinear Equations and Their Applications to Gas Dynamics [in Russian], Moscow (1978).
13. E. Kamke , Handbook of Ordinary Differential Equations [in Russian], Moscow (1971).
14. B. B. Boiko, A. I. Akimov, L. L. Vasil'ev, et al., Inzh.-Fiz. Zh., **58**, No. 5, 709-714 (1990).
15. N. V. Butenin, Yu. I. Neimark, and N. A. Fufaev, Introduction to the Theory of Nonlinear Vibrations [in Russian], Moscow (1987).
16. M. A. Gol'dshtik and V. N. Shtern, Izv. Akad. Nauk SSSR, Mech. Zhidk. Gaza, No. 2, 35-44 (1989).
17. Zh. Joss and D. Jozef, Elementary Theory of Stability and Bifurcations [Russian translation], Moscow (1983).